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# ACCELERATION WAVE IN A GAS—SOLID PARTICLE MIXTURE WITH CONSIDERATION OF FUSION

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A flow of gas mixed with solid particles occurs in many technological processes, in particular, in detonation deposition of finely dispersed metal particles on the surfaces of machine parts. The working substance (gas at high pressure and temperature) has sufficiently high state parameters so that fusion of the particles being driven occurs. This fusion may be of a nonequilibrium character, so that it is of interest to consider problems which develop in high velocity motion of such mixtures with consideration of this process.

The equations describing propagation of plane waves in an air-dispersed mixture of gas and solid particles at temperatures of the continuous phase sufficient for phase transition have the form [1, 2]

$$\partial x/\partial X = \rho_0 v,\ \partial\xi/\partial t = \varkappa = -(1/\tau)(\xi - \xi e),$$
$$\partial p/\partial X + \rho_0 \partial u/\partial t = 0,\ \partial e/\partial t + p\partial v/\partial t = 0,$$
$$e = e(S,\ v,\ \xi),\ p = -e_v\,(S,\ v,\ \xi),\ T = e_S\,(S,\ v,\ \xi),$$

where the Cartesian component x describing the motion of the medium is a function of the position of a point at the initial moment X and the current time t, i.e., x = x(X, t); $\rho_0$ is the initial density of the mixture; v, p, u, e, T, S are the specific volume, pressure, velocity, internal energy, temperature, and entropy of the mixture; $\xi$ is the relative mass concentration of the liquid phase; $\xi_e = \xi_e(S, v)$ is the equation of equilibrium fusion; $\tau$ is the relaxation time of the fusion process.

We will assume that at the initial moment the mixture has the following parameter values

$$v = v_0,\ \xi = \xi_0,\ S = S_0,\ x = X.$$

Following [3], we will define a second-order wave as a singularity in the flow propagating along the line y = y(Y, T) on which x(X, t) may have discontinuities in its second derivatives, while x(X, t), S(X, t), $\xi$(X, t) have continuous first derivatives. Second-order waves are called acceleration waves.

Thus, by definition, in an acceleration wave the equations

$$[x] = [v] = [u] = [S] = [\xi] = 0,\ [\varphi] = \varphi_1 - \varphi_2$$

are satisfied.

Using the equation of state and the kinetic and energy equations, we find

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$$[p] = [T] = [e] = [\dot{\xi}] = [\dot{S}] = 0, \tag{1}$$

and applying Maxwell's theorem, obtain

$$[\xi_x] = [S_x] = 0. \tag{2}$$

We introduce the quantity $a = [\dot{u}]$, describing the behavior of the wave which propagates at sonic velocity $v^2 = -v_0^2 \partial p/\partial v$. As was shown in [3], on the basis of Eqs. (1), (2), the equation describing the behavior of the parameter a,

$$da/dt = -\mu a + \mu a^2/\lambda \tag{3}$$

must satisfy the Cauchy condition

$$a(0) = a_0. \tag{4}$$

Here $\mu = v^2 p_\xi \varkappa_v / 2c_f^2$; $\lambda = p_\xi \varkappa_v c_f / v p_{vv}$; $c_f$ is the frozen speed of sound: the values of $\mu$, $\lambda$ are taken in the unperturbed state. In view of the fact that $\mu = v^2 e_{v\xi}^2 / (\tau 2 c_f^2 e_{\xi\xi})$ and the condition of thermodynamic stability $e_{\xi\xi} \geq 0$ it is evident that $\mu \geq 0$. Then the sign of $\lambda$ is determined by the sign of the expression $p_{vv} = \gamma(1+\gamma)p_{vv}^{2} > 0$, i.e., $\lambda > 0$.

We write the solution of the Cauchy problem, Eqs. (3), (4), in the form

$$a(t) = \lambda a_0 / [(\lambda - a_0) e^{\mu t} + a_0], \tag{5}$$

trivial analysis of which permits the following:

Conclusion 1. If the initial acceleration of the piston $a_0$ which sets in motion the mixture of gas and particles in which the nonequilibrium fusion occurs is such that $0 < a_0 < \lambda$, then the amplitude of the acceleration wave $a(t) \to 0$ with increase in t; if $\lambda = a_0$, then $a(t) \equiv a_0$, and for $a_0 > \lambda$ there exists a $t = t_* = -(1/\mu)\ln(1 - \lambda/a_0)$, then a(t) increases without limit, i.e., a shock wave is formed.

We will consider the near-frozen approximation in Eqs. (3), (4). We use the notation $\mu = \mu_1/\tau$, $\lambda = \lambda_1/\tau$, then as $\tau \to \infty$ Eq. (3) transforms to the equation $da_\infty/dt = \mu_1 a_\infty^2/\lambda_1$ [3], which has a solution of the Cauchy problem Eqs. (3), (4) $a_\infty = a_0/(1 - (\mu_1/\lambda_1)a_0 t)$. It is evident that in the approximation of mixture flow frozen with respect to fusion a shock wave is always formed and $t_* = \lambda_1/(\mu_1 a_0)$. Expanding solution (5) in a series in $\tau^{-1}$ for the condition $t/\tau \ll 1$, we obtain outside the boundary layer the solution

$$a(t) = a_\infty(t)(1 - \mu_1/a_\infty(t) t/\tau + o(t/\tau)^2),$$

which describes near-frozen flow to the accuracy of $o(\tau^{-2})$.

Choosing the equation of state in the form $e = \bar{c}_{V_1} T + L\xi$ [2], we find

$$\lambda = h^2 \frac{w}{v} \frac{c_f}{\gamma\tau} g(L_1), \quad L_1 = \frac{L}{c_{V_1} T}, \quad w = v - \beta_2$$

$$\mu = \frac{\gamma - 1}{2\gamma} \frac{1}{\tau} g(L_1), \quad g(L_1) = \frac{L_1^2}{L_1^2 + \xi^{-1}}, \quad \beta = \frac{\alpha}{r}, \quad h^2 = \frac{\gamma - 1}{\gamma + 1}.$$

In performing numerical calculations with Eq. (5) the effect of volume particle concentration $\eta = (m_2 + m_3)_0$ on the value of the critical acceleration $\lambda$ was studied. It proves to be the case (Fig. 1) that there exists some limiting value of $\eta = \eta_*$ was studied. It proves to be the case (Fig. 1) that there exists some limiting value of $\eta = \eta_*$, at which $\lambda$ reaches a maximum. Decrease in the number of particles leads to a decrease in the limiting acceleration. This is caused by the fact that the change $\eta \to 0$ implies that the mixture to a great extent has the properties of a pure gas, in which shock waves are formed without limitations as to limiting acceleration [3]. It can be shown that $\lambda \sim h^2 (c_f/\gamma\tau) r L_1^2 \eta$ at small $\eta$, $0 < L < \infty$, i.e., $\lambda$ is determined mainly by the fraction of particles in the mixture.

Increase in the quantity of particles $\eta > \eta_*$ also leads to a decrease in $\lambda$, caused by the smallness of the specific particle volume $1/r \ll 1$. Thus, the following situation exists within the mixture. At low particle concentrations the main process determining the limiting acceleration is the phase transition. With increase in $\eta \geq \eta_*$ the effect of the fusion process

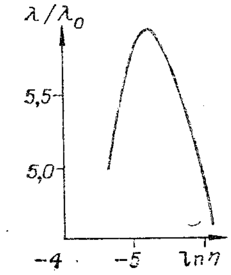


Fig. 1

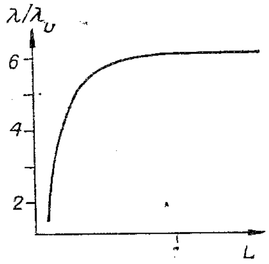


Fig. 2

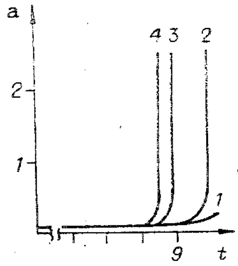


Fig. 3

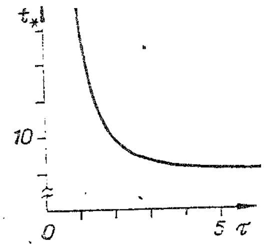


Fig. 4

decreases, and the effect of the volume fraction of particles (decrease in specific volume of the mixture) increases.

The effect of the heat of phase transition on λ was studied. It is obvious that there exists some limiting value of the heat of fusion $L_*$, after attainment of which the quantity λ will saturate, i.e., at $L > L_*$ the limiting acceleration changes little (Fig. 2). This is true because the limiting acceleration is determined by the difference $c_j^2 - c_e^2$, which depends only weakly on the heat of phase transition at high values $(\lambda \sim h^2(pv/\gamma\tau)\xi_0)$. With change in L over the interval $0 < L < L_*$ an abrupt rise in limiting acceleration occurs, caused by expenditure of gas energy in particle fusion. We note that at low $L_1$ $\lambda \sim h^2 \frac{pv}{\gamma\tau} \xi_0 L_1^2$, i.e., coincides with the asymptote for λ values corresponding to low particle concentrations. This permits study of highly concentrated air-dispersed media with a discrete phase having a sufficiently low heat of phase transition on the basis of low concentration gas–particle mixtures with a finite heat of phase transition.

Figure 3 shows the change in acceleration with time for several characteristic relaxation times (τ = 1, 2, 3, 4 for curves 1-4). It is evident that with increase in the relaxation time τ the mixture acceleration profile on the wave front approaches the acceleration profile in a mixture with frozen fusion process (see Eq. (5)). The shock wave formation time then decreases with increase in τ (Fig. 4), which is due to a decrease in the quantity of energy expended in the fusion process. It develops that at τ > 3 the phase transition has practically no effect on the shock-wave formation time.

In conclusion, we will note that the main calculation results were obtained with parameter values $c_{V1} = 750$, $c_{p1} = 1050$, $R = 300$, $T_0 = 2300$ K, $p_0 = 10^6$, $r = 2700$, $L = 2 \cdot 10^5$.

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